

$$\left(\int_{-\infty}^{\infty} e^{-x^2} dx \right)^2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} dx dy$$

$$= \int_0^{2\pi} \int_0^{\infty} e^{-r^2} r dr d\theta$$

$$= \int_0^{2\pi} \left(-\frac{e^{-r^2}}{2} \Big|_{r=0}^{r=\infty} \right) d\theta$$

$$= \pi$$

q.e.d.